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A Double-Ground-Plane Strip-Line System for Microwaves*

B. A. DAHLMAN†

Summary—**The double-ground-plane strip line consists of two parallel conducting planes with a conducting strip imbedded in a homogeneous dielectric between them. Transmission characteristics for this system are calculated, and design formula are given. Practical viewpoints on design and application of strip lines are discussed. System can be used as an inexpensive base for microwave circuits and is well adapted to laboratory experiments and mass production.**

LIST OF SYMBOLS

ϵ_0 = Absolute permittivity of free space, farads/m.
ϵ = Relative permittivity of dielectric in the line.
δ = Dielectric loss angle.
σ = Conductivity, mhos/m.
μ_0 = Absolute permeability of free space, henrys/m.
e = Base of natural logarithms.
λ_0 = Wavelength for transmission in free space, m.
λ = Wavelength along the line, m.
f = Frequency, cps.
Z_0 = Characteristic impedance, ohms.
C = Capacitance per unit length of the line, farads/m.
b = Width of strip, m.
$2h$ = Distance between ground planes, m.
d = Thickness of strip, m.
t = Width of ground planes, m.
E = Electric force, volts/m.
E_h = Homogeneous electric force far from the edge of a very wide strip, volts/m.
P_c = Power loss per unit length of the conductors, watts/m.
P_T = Power transmitted along the line, watts.
α_d = Dielectric attenuation, db/wavelength.
α_c = Conductor attenuation, db/m.
V = Potential difference between the ground planes and the strip, volts.

INTRODUCTION

DURING the last few years there has been considerable interest in new, simpler methods for manufacturing microwave circuits. In December, 1952, the Federal Telecommunication Laboratories presented an extensive report on a microwave printed-line system [1–3]. Although it is simple enough and has proved to be very useful for many circuits, it has the disadvantage of being an open system and is thus subject to some radiation. A shielded strip-line system was described by Barrett and Barnes, [4] but no real analysis was given.

The double-ground-plane strip line described in the paper has a thin strip of copper foil placed between two sheets of low-loss dielectric, and the outer sides of the dielectric sheets are covered with conducting planes.

The following analysis described provides a theory for the double-ground-plane strip line similar to that given in [2].

TRANSMISSION CHARACTERISTICS OF THE DOUBLE-GROUND-PLANE STRIP LINE

Interest is centered particularly on the phase wavelength, characteristic impedance and attenuation of the strip line.

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† Magnetic AB

Phase Wavelength

A cross-section of the line is shown in Fig. 1. Since the electric field does not extend outside the dielectric, the fundamental will be a pure TEM mode. The phase wavelength in the strip line is then $1/\sqrt{\epsilon}$ times the free-space wavelength, where ϵ is the dielectric constant of the material in the line:

$$\lambda = \lambda_0/\sqrt{\epsilon}. \tag{1}$$

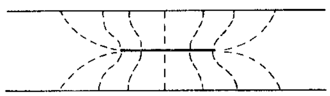

Fig. 1—Cross section of the double-ground-plane strip line.

Characteristic Impedance

For a TEM mode the characteristic impedance, Z_0, is given by

$$Z_0 = \sqrt{(\mu\epsilon_0\epsilon)}/C \tag{2}$$

where C is the capacitance per unit length. This capacitance is most easily calculated by the conformal mapping method.

An infinitely thin strip is assumed. As the subsequent analysis will show, the electric strength between the strip and the conducting planes is very nearly equal to the homogeneous electric strength between two parallel conducting planes of infinite width with the same potential difference and the same spacing, except very close to the strip edges. We can therefore, to a good approximation, calculate the field at the edge of the strip by assuming that the strip extends to infinity in one direction. Figs. 2(a) and 2(b) show the actual strip and the approximation. The coordinate axes of a rectangular coordinate system in the z plane are shown in Fig. 2(b).

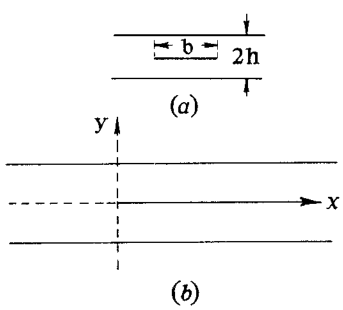

Fig. 2—The actual strip and the approximation.

By means of the transformation

$$z = \frac{2h}{\pi}\log z_1 \qquad z_1 = x_1 + jy_1 \tag{3}$$

the strip is transformed to the x_1 axis from $x_1 = 1$ to $x_1 = \infty$ in the z_1 plane and the ground planes to the y_1 axis.

The z_1 plane is shown in Fig. 3. The field in the z_1 plane is given in several textbooks, the lines of force being ellipses, but to get a better idea of the problem we shall make one more transformation. This will prove to be useful later in the calculation of the attenuation.

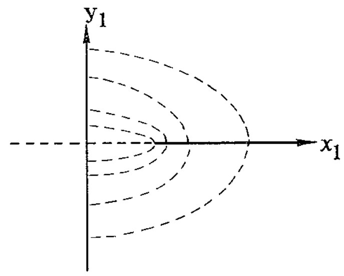

Fig. 3—The double-ground-plane-strip line transformed to the z plane.

The transformation

$$z_1 = \cos\omega \qquad \omega = u + jv \tag{4}$$

transforms the strip to the v axis and the ground planes to the line $u = \pi/2$ in the ω plane (see Fig. 4).

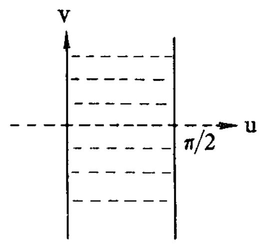

Fig. 4—The double-ground-plane-strip line transformed to the w plane.

The center line of the actual strip corresponds to the point $(b/2, \theta)$ in the z plane and to

$$\omega_p = u_p + jv_p$$

in the ω plane. ω_p is given by (3) and (4).

$$\left.\begin{aligned} u_p &= 0 \\ v_p &= \pm \operatorname{arc\,cosh} e^{\pi b/4h} \end{aligned}\right\} . \tag{5}$$

The capacitance per unit length between the v axis and the line $u = \pi/2$ from $v = -|v_p|$ to $v = |v_p|$ is

$$C = \epsilon\epsilon_0 \frac{\operatorname{arc\,cosh} e^{\pi b/4h}}{\pi} \times 4. \tag{6}$$

As this represents the capacitance of half of the strip, the impedance given by (2) is

$$Z_0 = \sqrt{(\mu_0\epsilon\epsilon_0)}/8 \times \epsilon\epsilon_0 \frac{\operatorname{arc\,cosh} e^{\pi b/4h}}{\pi} \cdot \tag{7}$$

In Fig. 5, Z_0 is plotted aganst h/b. An approximate expression for the impedance of a strip of finite thickness d can easily be obtained by assuming that the image of the strip in the ω plane is the line $u = \Delta u$ (see Fig. 6).

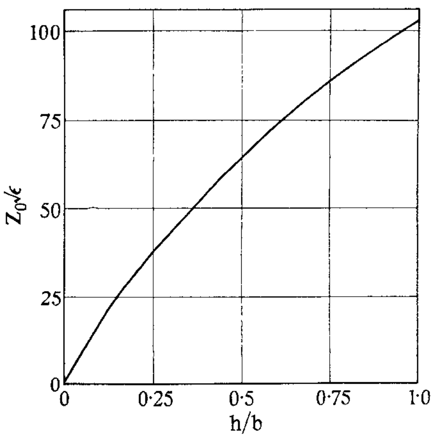

Fig. 5—Characteristic impedance of double-ground-plane strip line.

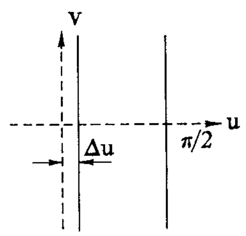

Fig. 6—Image of strip of finite thickness.

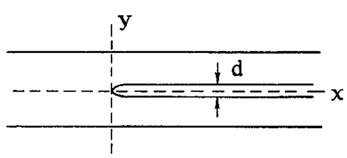

Fig. 7—Strip of finite thickness.

The image of this contour in the z plane is shown in Fig. 7. The assumption will be made that $d \ll h$. The equation for the strip contour in the z plane given by (3) and (4) is then

$$y^2 = \left(\frac{d}{2}\right)^2 \times (1 - e^{-x\pi/h}) \tag{8}$$

and the value of Δu is

$$\Delta u = \frac{\pi}{4} \times \frac{d}{h}\cdot \tag{9}$$

For thin strips the approximation for the actual strip cross section obtained in this way is probably as good as any. To get the impedance of a strip of finite thickness we should multiply the value of the impedance obtained from (7) and Fig. 5 by a factor

$$D_z = 1 - \frac{d}{2h} \tag{10}$$

The approximation entailed in assuming that the strip extends to infinity in one direction will next be checked. If the electric field in the ω plane is $E(\omega)$ the field in the z plane is given by

$$E(z) = E(\omega) \times \overline{d\omega/dz} \tag{11}$$

where the bar indicates the complex conjugate.

Thus, using (3) and (4),

$$\begin{aligned} E(z) &= E(\omega)\, \overline{\frac{d}{dz}\,(\text{arc cos } e^{\pi z/2h})} \\ &= E(\omega)\, \frac{\pi}{2h\sqrt{(e^{-zh/\pi} - 1)}}\cdot \end{aligned} \tag{12}$$

The ratio between the actual field strength at the surface of the strip and the homogeneous field strength far from the edge is plotted in Fig. 8. It is thus obvious that for $b/h > 1.5$ the approximation is accurate to within a few per cent. Even for a ratio of $b:h$ as low as unity, the value of Z_0 given in (7) and Fig. 5 is only 3 per cent bigger than the exact value calculated in [11].

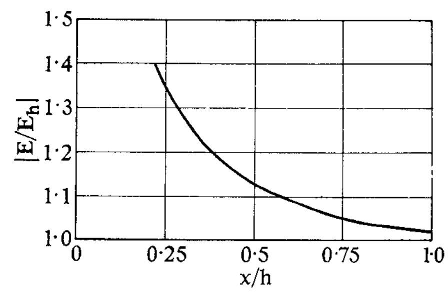

Fig. 8—Variation of the electric field across the strip.

Attenuation

The attenuation is due to dielectric losses and losses in the conductors. For TEM-mode propagation the attenuation due to dielectric losses is given by the following expression:

$$\alpha_d = 20 \times \frac{\delta\pi}{\log 10} = 27\cdot 2\delta \text{ decibels per wavelength.} \tag{13}$$

The conductor losses per unit length of the line are

$$P_c = k\int |E(z)|^2 |dz| = k\int |E(\omega)|^2 \left|\frac{d\omega}{dz}\right| |d\omega| \tag{14}$$

where

$$k = \sqrt{\left(\frac{\pi f}{\sigma\mu_0}\right)} \times \frac{\epsilon\epsilon_0}{2}\cdot \tag{15}$$

The integral should be taken along the conductor contours in the z or ω plane. The approximation for an actual strip of thickness d is used as described in the previous section and shown in Fig. 8. The integration is made in the Appendix.

$$P_c = \frac{4V^2k}{h}\left(\frac{b}{h} + \frac{2}{\pi}\log\frac{4h}{\pi d}\right) \quad (16)$$

The power transmitted along the line is

$$P_T = \frac{V^2}{2Z_0}$$

Attenuation per unit length due to conductor losses is

$$\alpha_c = \frac{10}{\log 10} \times \frac{P_c}{P_T} = \frac{80Z_0k}{2 \cdot 3h}\left(\frac{b}{h} + \frac{2}{\pi}\log\frac{4h}{\pi d}\right). \quad (17)$$

The value of k obtained from (15) is substituted in (17) and we obtain

$$\alpha_c\sqrt{\frac{\sigma}{f\epsilon\epsilon_0}}\, h = 0 \cdot 082 Z_0\sqrt{\epsilon}\left(\frac{b}{h} + \frac{2}{\pi}\log\frac{4h}{\pi d}\right). \quad (18)$$

Eq. (18) is illustrated in Fig. 9.

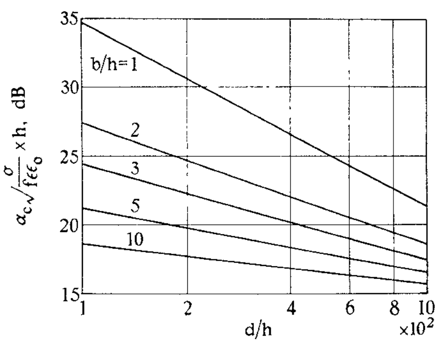

Fig. 9—Attenuation due to conductor losses.

A 50-ohm strip line with polystyrene dielectric at a frequency of 3,000 mc is considered:

$$\alpha_d = 27.2 \times 0.001 = 0.027 \text{ db/wavelength}$$

$$\lambda = 10/\sqrt{2.5} = 6.3 \text{ cm}$$

If the conductor losses are to be smaller than the dielectric losses,

$$\alpha_c < 0.027/0.063 = 0.43 \text{ db/m}$$

$$\sqrt{\frac{\sigma}{f\epsilon\epsilon_0}} = 29{,}000$$

$$h/b = 0.65,\ b/h = 1.55,$$

If we choose

$$h = 2 \text{ mm}$$

$$\alpha_c\sqrt{\frac{\sigma}{f\epsilon\epsilon_0}} \times h = 25 \text{ db/m}$$

$$d/h = 0.03$$

$$d = 0.06 \text{ mm}$$

Practical Viewpoints on the Design and Application of Strip-Line Systems

Dimensions

Various factors such as impedance, the maximum permissible attenuation, power-handling capacity and available space determine the practical dimensions of strip-line systems. In the previous section, impedance and attenuation characteristics were considered. The power-handling capacity is determined by the power dissipation allowed in the line, and by the electric strength of the insulation.

The power dissipation per unit length of the line is

$$P_1 = P_T\alpha\frac{\log 10}{10} = 0.23 \times P_T\alpha \text{ watts per unit length} \quad (19)$$

where P_T is the transmitted power and α is the total attenuation in decibels per unit length [(13) and (18)].

The strongest electric field occurs at the strip edges. The position of the strip edge in the z plane (see Fig. 8) is

$$\left[\frac{-\pi}{h}\left(\frac{d}{4}\right)^2, 0\right],$$

which corresponds to the point $(\Delta u, 0)$ in the ω plane. From (12) the electric field at the edge is then

$$E_{\max} = \frac{4}{\pi}\frac{V}{d}. \quad (20)$$

Even though the calculations made so far are based on the assumption of ground planes with infinite extension, the results are valid with good approximation even for finite ground planes, so long as the electric field at the edge of the planes is small compared with the homogeneous field between the ground planes and the strip. The variation of the electric field across the ground planes, given by (12), is shown in Fig. 10.

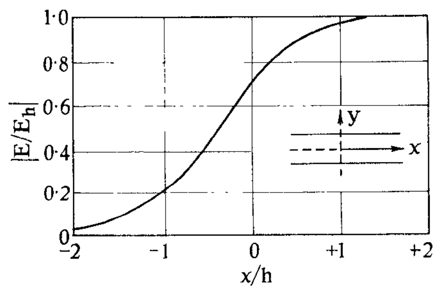

Fig. 10—Variation of the electric field across the ground planes.

A ground-plane width of

$$t = b + 4h$$

is adequate for most purposes.

Manufacturing Methods

Double-ground-plane strip-line systems can be manufactured by methods similar to those used in the technique described in [1] and [5]. For mass production the best method so far found is a process by which the strip configuration is printed on one side of a copper-clad dielectric sheet. The excess material is removed by an etching process. An unclad dielectric sheet of the same thickness is then laid on top of the strip and the sandwich is placed between two suitable ground-plane conductors. If one of the dielectric sheets is clad on both sides and the other on one side the copper skins may serve as ground planes.

For laboratory use a convenient way of making strip-line components is to cut out the desired strip configuration in copper foil and place it between two dielectric sheets.

Suitable Dielectrics

Low-loss copperclad dielectrics are commercially available. For laboratory purposes unclad polystyrene and Teflon sheets are often practicable.

Advantages of Strip-Line Systems

Microwave circuits in the conventional waveguide and coaxial systems are often mechanically complicated and rather expensive. This is especially true of networks with more than four terminals—the costs rise very rapidly with the number of terminals. For such types of circuits the strip-line technique can be used to reduce the costs considerably. In a practical example, Fig. 11 shows a feeder system for a certain antenna.

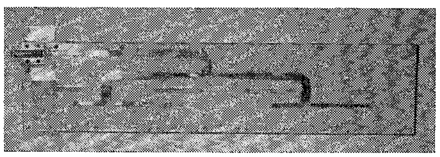

Fig. 11—An antenna feeder system in a strip line.

The whole system involving 50/25-ohm tapers, right-angle bends, tee junctions and an adaptor to a type-N female connector is matched throughout at about 3,000 mc. The dielectric is 2 mm thick polystyrene, which gives an attenuation of about 0.03 db per wavelength. The strip is 0.04 mm thick copper foil, which gives a conductor attenuation of about 0.5 db/m. The whole system has an attenuation of about 0.3 db. If the circuit were to be constructed in a coaxial system the circuit configuration would probably be cut out in two solid metal blocks with a milling machine. The two blocks put together would make the outer conductor. The inner conductor would have to be supported by some dielectric supports which should be compensated. The long tapers which are easily made in the strip system would probably have to be substituted by transformers. An attempt has been made to estimate the production costs for the two systems. The comparison is given in Table I.

TABLE I

Number of units	Price per unit	
	Coaxial line	Strip line
1	£15	£2
25	£5	15s.
1,000	10s.	2s.

The costs for the strip-line units should be 10–20 per cent of those of the coaxial units.

Conclusions

The double-ground-plane strip line is a useful complement to the conventional waveguide and coaxial systems and to other types of strip lines. Complicated networks can be built in this system at much lower cost than in the more conventional waveguide and coaxial systems and they do not have the disadvantages of unshielded systems.

Appendix

Evaluation of the Integral of (14)

Integration is carried out along the conductor contours in the ω plane.

$$P_c = k \int \left| E(\omega) \right|^2 \left| \frac{d\omega}{dz} \right| \left| d\omega \right|$$

$$\left| E(\omega) \right| \simeq \frac{2V}{\pi}$$

where V is the amplitude of the voltage between the strip and the ground planes.

$$\left| \frac{d\omega}{dz} \right| = \frac{\pi}{2h} \left| \cot \omega \right|$$

If the integral is taken along the conductor image lines from $v=0$ to $v=v_p$ we obtain $\frac{1}{4}$ of the total losses.

Losses in the Ground Planes

Integration is carried out along $u=\pi/2$

$$\left| \cot \omega \right| = \left| \tanh v \right|$$

$$P_{c1} = 4k \frac{2V^2}{\pi h} \int_0^{vp} \tanh v dv = \frac{V^2 2bk}{h^2} \tag{19}$$

v_p is given by (5).

Losses in the Strip

Integration is carried out along $u=\Delta u$

$$\left| \cot \omega \right| = \frac{1}{\sqrt{[(\Delta u)^2 + \tanh^2 v]}}$$

a) $$v = 0 —— n\,\Delta u \qquad 1 \ll n \ll \frac{1}{\Delta u}$$

$$P_{c2a} = 4k\frac{2V^2}{\pi h}\int_0^{n\Delta u}\frac{dv}{\sqrt{[(\Delta u)^2 + v^2]}} = \frac{8V^2k}{\pi h}\log 2n \quad (20)$$

b) $$v = n\Delta u \text{ —— } v_p$$

$$P_{c2b} = 4k\frac{2V^2}{\pi h}\int_{n\Delta u}^{v_p}\frac{dv}{\tanh v}$$

$$= \frac{8V^2k}{\pi h}\left(\frac{\pi b}{4h} - 1 - \log n - \log \Delta u\right) \quad (21)$$

v_p given by (5).

The sum of (19), (20) and (21) is

$$P_c \cong \frac{4V^2k}{\pi h}\left(\frac{b}{h} + \frac{2}{\pi}\log\frac{4h}{\pi d}\right).$$

Acknowledgment

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Correspondence

Transmission Characteristics of Sandwiches*

The radome design equations which are in common use are based on the transmission characteristics of sandwiches. Transmission line techniques provide a convenient method for determining these characteristics. The transmission line analogy of wave propagation has been discussed by Ramo and Whinnery.[1] A restricted application of this analogy is given in a report by Snow.[2] In this note, a more general application will be discussed.

The sandwich shown in Fig. 1 will be considered. Although this sandwich has only three layers, this does not indicate that the procedure is limited to any maximum number of sheets. It is assumed that the sandwich consists of plane, homogeneous, isotropic sheets of infinite extent. The sheets may be lossy. (A lossy dielectric can be characterized by either the dielectric constant or permeability or both being complex.) The wave incident upon the sandwich is plane and linearly polarized.

Since the transmission characteristics are different when the polarization is perpendicular and parallel to the plane of incidence, it is necessary to consider these two polarizations separately. When the polarization of the wave is neither perpendicular nor parallel to the plane of incidence, it is necessary to divide the wave into perpendicular and parallel components. The subscripts

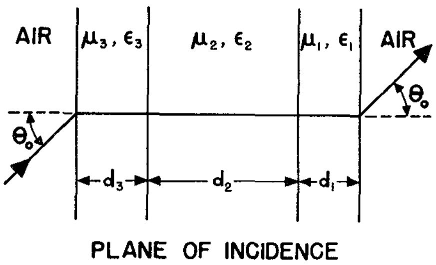

Fig. 1—A three-layer sandwich.

"⊥" and "‖" denote components and parameters associated with the components of the wave whose polarizations are perpendicular and parallel, respectively, to the plane of incidence.

The ith sheet is analogous to a section of transmission line whose length is d_i, whose propagation constant is

$$\gamma_i = \alpha_i + j\beta_i = j(2\pi/\lambda_0)\sqrt{\mu_i\epsilon_i - \sin^2\theta_0}, \quad (1)$$

and whose characteristic impedance is

$$\eta_{i\perp} = \mu_i/\sqrt{\mu_i\epsilon_i - \sin^2\theta_0} \quad (2)$$

or

$$\eta_{i\parallel} = \sqrt{\mu_i\epsilon_i - \sin^2\theta_0}/\epsilon_i, \quad (3)$$

where λ_0 is the wavelength in air. In general, $\eta_{i\perp}$ and $\eta_{i\parallel}$ are complex. The incident electric field E_4^+ required to produce a unit field at the opposite side of the sandwich is given by the equation

$$E_4^+ = \left(\frac{1+\Gamma_0'}{1+\Gamma_1}\right)\left(\frac{1+\Gamma_1'}{1+\Gamma_2}\right)\left(\frac{1+\Gamma_2'}{1+\Gamma_3}\right)\cdot\left(\frac{1+\Gamma_3'}{1+\Gamma_4}\right)e^{\gamma_1 d_1+\gamma_2 d_2+\gamma_3 d_3}, \quad (4)$$

where

$$\Gamma_0' = 0, \quad (5)$$

$$\Gamma_i' = \Gamma_i e^{-2\gamma_i d_i}, \quad (6)$$

$$\Gamma_{i+1} = \frac{\eta_i(1+\Gamma_i') - \eta_{i+1}(1-\Gamma_i')}{\eta_i(1+\Gamma_i') + \eta_{i+1}(1-\Gamma_i')}. \quad (7)$$

Transmission line charts, i.e., Smith ($R-X$) and Carter ($Z-\theta$) charts, provide a convenient method for determining E_4^+.

The voltage reflection coefficient is Γ_4 and the power reflection coefficient is $|\Gamma_4|^2$. The ratio of power transmitted to power incident is $1/|E_4^+|^2$. The increase in phase retardation due to the sandwich is

$$\Phi = (\text{angle of } E_4^+) - (360°/\lambda_0)(d_1 + d_2 + d_3)\cos\theta_0. \quad (8)$$

* Presented at the Symposium on Antennas and Radomes in Tracking Systems, University of Vermont, Burlington, Vermont, June 9 and 10, 1955.

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